Modelling and numerical aspects of fluid-saturated granular materials: application to shear and chute flows.

C. Varsakelis & M. V. Papalexandris

Institute of Mechanics, Materials and Civil Engineering,
Université Catholique de Louvain,
B-1348, Louvain la Neuve,
Belgium

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Physical assumptions

We consider a saturated mixture of an isotropic granular material and a simple fluid.

i) Starting point: a continuum model for granular mixtures.
(M.V. Papalexandris (2004), *J. Fluid. Mech.*, **517**).

- Each phase is treated as an open thermodynamic system that interacts with the other one.

ii) Low-Mach number approximation.
(C. Varsakelis, M.V. Papalexandris (2011), *J. Fluid. Mech.*, **669**).

iii) For constant densities the low-Mach number equations reduce to...

Governing equations for constant density flows I

Granular phase

$$\nabla \cdot \boldsymbol{u}_s = 0\,,$$

$$\rho_s \phi_s \frac{\mathrm{d}\boldsymbol{u}_s}{\mathrm{d}t_s} + \nabla(p_s \phi_s) = \frac{1}{Re} \nabla \cdot (\phi_s \, \mu_s \, \boldsymbol{V}_s^v) - \nabla \cdot (\Gamma_s \nabla \phi_s \otimes \nabla \phi_s) + p_f \nabla \phi_s + \delta(\boldsymbol{u}_f - \boldsymbol{u}_s) + \rho_s \phi_s \mathbf{g}\,.$$

Fluid phase

$$\nabla \cdot ((\boldsymbol{u}_s - \boldsymbol{u}_f)\phi_f) = 0\,,$$

$$\rho_f \phi_f \frac{\mathrm{d}\boldsymbol{u}_f}{\mathrm{d}t_f} + \nabla p_f \phi_f = \frac{1}{Re} \nabla \cdot (\phi_f \mu_f \, \boldsymbol{V}_f^v) - p_f \nabla \phi_s - \delta(\boldsymbol{u}_f - \boldsymbol{u}_s) + \rho_f \phi_f \mathbf{g}\,.$$

Compaction equation for the solid volume fraction:

$$\frac{\mathrm{d}\phi_s}{\mathrm{d}t_s} = 0\,.$$

Governing equations for constant density flows II

- $\boldsymbol{\sigma} = \nabla \cdot (\Gamma_s \nabla\phi_s \otimes \nabla\phi_s)$: configuration stress tensor; shear stresses at zero shear rates.
- $p_f \nabla\phi_s + \delta(\boldsymbol{u}_f - \boldsymbol{u}_s)$ models momentum exchanges between the two phases. $p_f \nabla\phi_s$ is a non-conservative product.
- δ: interphasial drag coefficient.

The numerical algorithm I

Prediction stage

i) Integrate the compaction equation for ϕ_s with the Corner Transport Upwind scheme.
(P. Colella, (1990), *J. Comput. Phys.* **87**).

ii) Check for interfaces and perform a regularization for ϕ_s in the vicinity of the interface points.

iii) **Granular phase**

- **(a)** Momentum equation: time integration with Adams–Bashforth scheme (Crank-Nicolson for the viscous fluxes).
- **(b)** Projection method for the pressure p_s.
- **(c)** A Poisson equation is solved at each step.

The numerical algorithm II

iv) Fluid phase

- **(a)** Momentum equation: time integration with Adams Bashforth scheme.
- **(b)** Projection method for the pressure p_f.
- **(c)** A second-order elliptic PDE is solved at each step.

Corrector stage

The flow-chart of the correction stage is similar to that of the prediction stage.

Sand-water mixture: physical parameters

Granular material (sand)

- $\rho_s = 2200\ kg/m^3$.
- $\Gamma_s = k_2 \rho_s \phi_s, \qquad k_2 = 4 \times 10^{-8}\ m^4/s^2$.

  (S.L. Passman et al., (1986), *J. Rheol.*, **30**).
- $\mu_s = \overline{\mu}_s \frac{\phi_s^2}{(1-\phi_s)^2}, \qquad \overline{\mu}_s = 723\ kg/ms$.

  (S.B. Savage, (1979), *J. Fluid. Mech.*, **92**).

Interstitial fluid (water)

- $\rho_f = 1000\ kg/m^3$.
- $\mu_f = 1 \times 10^{-3}\ kg/ms$.

Shear flow for water-sand mixture

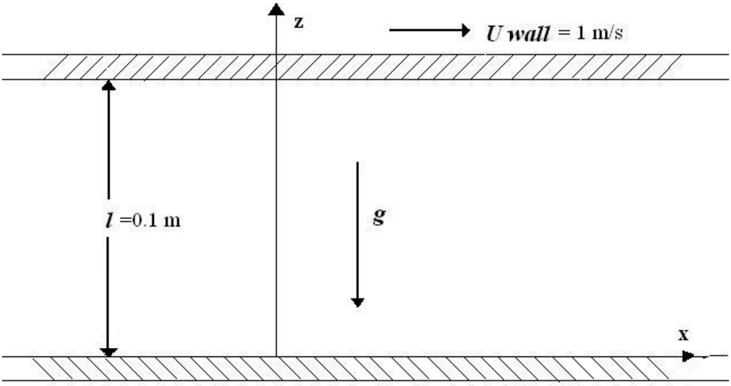


Figure : Geometry of simple shear flow.

Initial configuration of the mixture

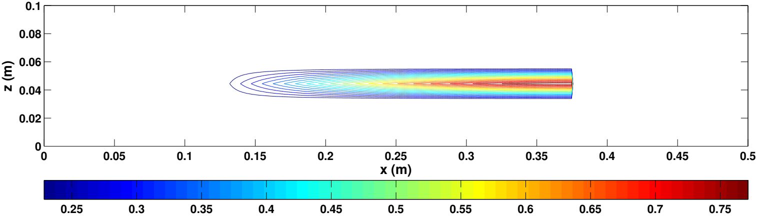


Figure : A granular bulb of high-particle concentration is placed between the plates and the rest of the domain is filled with a dilute water-sand mixture. Initially, the mixture is at rest.

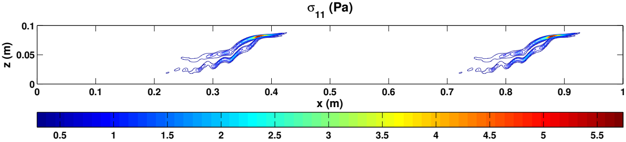
Volume fraction evolution without gravity

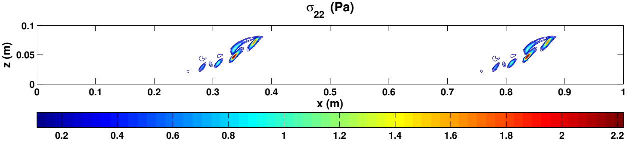
Figure : Evolution of ϕ_s, time interval $0 - 13$ s.

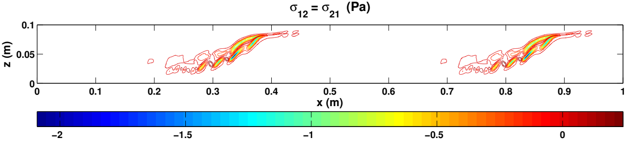
Volume fraction evolution with gravity

Figure : Evolution of ϕ_s, time interval $0 - 3.5$ s.

Configuration stresses: $t = 0.5\ s$

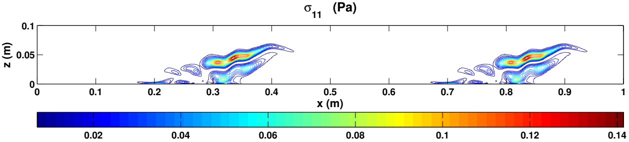


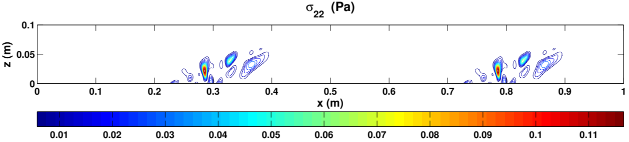


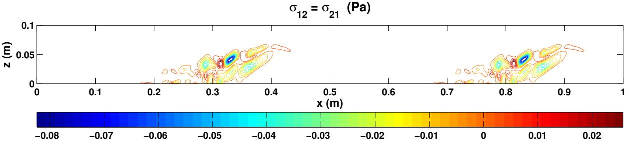


Numerical results

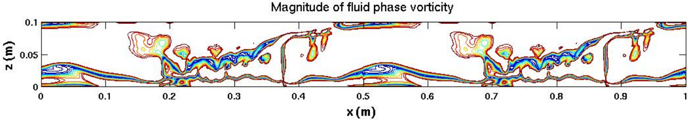
Configuration stresses: $t = 3.5\ s$

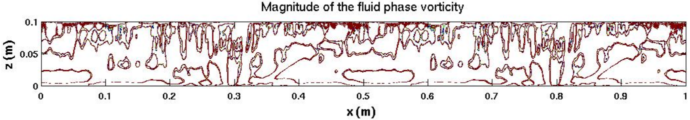






Vorticity of the fluid phase





- Gravity driven flow of a fluid–saturated granular layer down an inclined plane.
- Mixture of water with beach sand.
- Angle of inclination: 30^o.
- Flow susceptible to Kapitza-type instability: initial condition chosen so as to trigger instability.

Volume fraction evolution

Figure : Evolution of ϕ_s, time interval $0 - 2.5$ s.

Thank you!